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A Standby System with Priority to Repair over Preventive Maintenance

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Abstract: ***A cold standby system of two identical units is analyzed stochastically under priority in repair disciplines. Unit has a constant failure from normal mode. There is a single server who visits the system immediately to carry out repair activities. Server conducts preventive maintenance of the unit after a pre-specific time't' of operation. However, repair of the unit is done by the server at its failure. Priority is given to repair of one unit over preventive maintenance of the other unit. The random variables associated with different rates are statistically independent. The failure time and time by which unit goes for preventive maintenance follow exponential distribution while the distributions of maintenance and repair times are taken as arbitrary with different probability density functions. The expressions for several reliability measures of the system are derived using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behavior of MTSF, availability and profit function for particular values of various parameters and costs.***

Key words: Cold standby system / Maximum operation time / Preventive maintenance / Repair / Priority / and Stochastic Analysis.

Introduction

The performance of operating systems can be improved by providing redundancy as well as by conducting maintenance and repair of the failed components. And, cold standby systems are one of the most important structures in reliability engineering. Therefore, a lot of research work has been done on reliability modeling of cold standby systems considering different aspects on failure and repair policies. Gopalan and Naidu (1982), Cao and Wu (1989), Lam (1997) and Yadavalli et al. (2004) developed reliability models of a cold standby system with single repair facility.

Further, continued operation and ageing of operable systems reduce their performance, reliability and safety. Therefore, preventive maintenance of such systems can be conducted after a pre-specific time (called maximum operation time) in order to slow the deterioration process. Recently, Malik (2013) investigated a reliability model of a computer system with cold standby redundancy and preventive maintenance. Also, sometimes it becomes necessary to give priority in repair disciplines of one unit over the other in order to reduce down time of the system. Chhillar et al. (2013) analyzed a parallel system with priority to repair over maintenance subject to random shocks.

In view of the above practical situations in mind, here a reliability model for a two-unit cold standby system is developed considering the aspect of priority in repair disciplines. The units are identical in nature which may fail directly from normal mode. There is a single server who visits the system immediately for conducting maintenance and repair. Server conducts preventive maintenance of the unit after a maximum operation time't'. However, repair of the unit is done at its complete failure. Priority is given to repair of one unit over maintenance of the other unit. The random variables associated with failure time, completion of maximum operation time, preventive maintenance and repair times are statistically independent. The failure time and time by which unit goes for preventive maintenance follow exponential distribution while the distributions for maintenance and repair times are taken as arbitrary. Several measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to maintenance and repair, expected number of maintenances and repairs of the unit and profit function are obtained using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behavior of MTSF, availability and profit function for particular values of various parameters and costs.

Methodology

The semi-Markov and regenerative point processes are adopted to obtain reliability measures of the system model. A brief description as follows:

Semi-Markov Process

It is a continuous-time stochastic process in which transition from one state to another is governed by the transition probabilities of a Markov process and time interval between two successive transitions is a random variable whose distribution may depend upon the state from which transition takes place as well as on the state to which next transitions take place.

Regenerative Process

It is a stochastic process with time points at which, from a probabilistic point of view, the process restarts itself. These time points may themselves be determined by the evolution of the process. i.e., the process $\{X(t),\ t \geq 0\}$ is a regenerative process if there exist time points $0 \leq T_0 < T_1 < T_2 <$ ... such that the post-T_k process $\{X(T_{k+t}\ ) : t \geq 0\}$ has the same distribution as the post-T process $\{X(T0 + t) : t \geq 0\}$ and is independent of the pre-T_k process $\{X(t) : 0 \leq t < T_k\}$. These time points are called regenerative points. Regenerative stochastic process was defined by Smith (1955) and has been crucial in the analysis of the complex system.

Notations and Assumptions

E	: Set of regenerative states.
O/Cs	: The unit is operative/cold standby
α_0	: The constant rate by which unit undergoes for preventive maintenance
λ	: Constant failure rate of the unit.
f(t)/F(t)	: pdf /cdf of preventive maintenance time
g(t)/G(t)	: pdf /cdf of repair time of a failed unit
P_m/WP_m /PM	:The unit is under preventive

maintenance/waiting for preventive Maintenance / under preventive maintenance continuously from previous state.

$FU_r/Fw_r/FUR$:The failed unit is under repair/waiting for repair/under repair continuously from previous state.

$q_{ij}(t)$ / $Q_{ij}(t)$:pdf and cdf of direct transition time from a regenerative state S_i to a regenerative state S_j without visiting any other regenerative state

$q_{ij.k}(t)$ / $Q_{ij.k}(t)$:pdf and cdf of first passage time from a regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state k once in (0,t].

m_{ij} : The contribution to mean sojourn time in regenerative state S_i when system is to make transition in to regenerative state S_j . Mathematically, it can be written as

$$m_{ij} = E(T_{ij}) = \int_0^\infty t d[Q_{ij}(t)] = -q_{ij}^{*'}(0),$$

where T_{ij} is the transition time from state S_i to S_j; S_i , S_j ε E.

μ_i : The mean Sojourn time in state S_i this is given by

$$\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$$, where T_i is the sojourn time in state S_i.

$W_i(t)$: Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more regenerative states.

⊗/⊕ :Symbol for Laplace Stieltjes convolution/Laplace convolution

~/* : Symbol for Laplace Steltjes transform (LST)/ Laplace transform (LT)

The possible transition diagram of the system model is shown in fig. 1.

State Transition Diagram

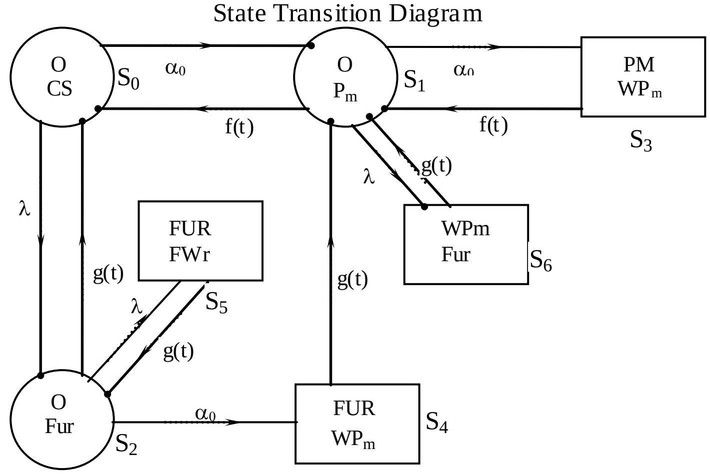

Fig 1.

Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt \tag{1}$$

$$p_{01} = \frac{\alpha_0}{\alpha_0+\lambda},\; p_{02} = \frac{\lambda}{\alpha_0+\lambda},\; p_{10} = f^*(\alpha_0+\lambda),$$

$$p_{13} = \frac{\alpha_0}{\alpha_0+\lambda}[1-f^*(\alpha_0+\lambda)]$$

$$p_{16} = \frac{\lambda}{\alpha_0+\lambda}[1-f^*(\alpha_0+\lambda)],\; p_{20} = g^*(\alpha_0+\lambda),$$

$$p_{25} = \frac{\lambda}{\alpha_0+\lambda}[1-g^*(\alpha_0+\lambda)]$$

$$p_{24} = \frac{\alpha_0}{\alpha_0+\lambda}[1-g^*(\alpha_0+\lambda)],$$

$$p_{31} = p_{41} = p_{52} = p_{61} = 1$$

$$p_{11.3} = \frac{\alpha_0}{\alpha_0+\lambda}[1-f^*(\alpha_0+\lambda)],$$

$$p_{21.4} = \frac{\alpha_0}{\alpha_0+\lambda}[1-g^*(\alpha_0+\lambda)],$$

$$p_{22.5} = \frac{\lambda}{\alpha_0+\lambda}[1-g^*(\alpha_0+\lambda)] \tag{2}$$

It can be easily verified that

$$p_{01}+p_{02} = p_{10}+p_{13}+p_{16} = p_{20}+p_{24}+p_{25} = 1 \tag{3}$$

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0 = \frac{1}{\alpha_0+\lambda},\; \mu_1 = \frac{1}{\alpha_0+\lambda}[1-f^*(\alpha_0+\lambda)],$$

$$\mu_2 = \frac{1}{\alpha_0+\lambda}[1-g^*(\alpha_0+\lambda)]$$

$$\mu_1' = \frac{\alpha_0+\alpha}{\alpha(\alpha_0+\alpha+\lambda)},\; \mu_2' = \frac{1}{\theta} \text{ and } \mu_6 = \frac{1}{\theta} \tag{4}$$

Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for

$$\phi_i(t): \quad \begin{aligned} \phi_0(t) &= Q_{01}(t)\otimes\phi_1(t) + Q_{02}(t)\otimes\phi_2(t) \\ \phi_1(t) &= Q_{10}(t)\otimes\phi_0(t) + Q_{13}(t) + Q_{16}(t) \\ \phi_2(t) &= Q_{20}(t)\otimes\phi_0(t) + Q_{24}(t) + Q_{25}(t) \end{aligned} \tag{5}$$

Taking LST of relations (5) and solving for $\tilde{\varphi}_0(s)$. We have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \tag{6}$$

The reliability of the system model can be obtained by taking inverse Laplace transformation of (6). The mean time to system failure (MTSF) is given by

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$$MTSF = \lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{\mu_0+\mu_1 p_{01}+\mu_2 p_{02}}{1-p_{01}p_{10}-p_{02}p_{20}} \quad (7)$$

Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are given as

$$A_0(t) = M_0(t) + q_{01}(t)\oplus A_1(t) + q_{02}(t)\oplus A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)\oplus A_0(t) + q_{11.3}(t)\oplus A_1(t) + q_{16}(t)\oplus A_6(t)$$
$$A_2(t) = M_2(t) + q_{20}(t)\oplus A_0(t) + q_{21.4}(t)\oplus A_1(t) + q_{22.5}(t)\oplus A_2(t)$$
$$A_6(t) = q_{61}(t)\oplus A_1(t) \quad (8)$$

where $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\alpha_0+\lambda)t},\ M_1(t) = e^{-(\alpha_0+\lambda)t}\overline{F(t)},$$
$$M_2(t) = e^{-(\alpha_0+\lambda)t}\overline{G(t)} \quad (9)$$

Taking LT of relations (8) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N}{D},\ \text{where} \quad (10)$$

$$N = \mu_0\{(1-p_{22.5})[(1-p_{11.3})-p_{16}p_{61}]\}$$
$$+\mu_1\{p_{01}(1-p_{22.5})+p_{02}p_{21.4}\}+\mu_2\{p_{02}p_{61}-p_{02}(1-p_{11.3})\}$$

$$D = \mu_0\{(1-p_{11.3})(1-p_{22.5})-p_{61}p_{16}(1-p_{22.5})\}$$
$$+\mu_1'\{p_{01}(1-p_{22.5})+p_{02}p_{21.4}\}+\mu_2'\{p_{02}p_{61}p_{16}$$
$$+p_{02}(1-p_{11.3})\}+\mu_6'\{p_{02}p_{21.4}p_{16}+p_{01}(1-p_{22.5})\}$$

Busy Period Analysis for Server

Let $B_i^p(t)$ and $B_i^R(t)$ be the probability that the server is busy due to preventive maintenance and repair of the unit at an instant 't' given that system entered state S_i at t=0.

$$B_0^P(t) = q_{01}(t)\oplus B_1^p(t) + q_{02}(t)\oplus B_2^p(t)$$
$$B_1^P(t) = W_1(t) + q_{10}(t)\oplus B_0^p(t) + q_{11.3}(t)\oplus B_1^p(t) + q_{16}(t)\oplus B_6^p(t)$$
$$B_2^P(t) = q_{20}(t)\oplus B_0^p(t) + q_{22.5}(t)\oplus B_2^p(t) + q_{21.4}(t)\oplus B_1^p(t)$$
$$B_6^P(t) = q_{61}(t)\oplus B_1^p(t)$$

and

$$B_0^R(t) = q_{01}(t)\oplus B_1^R(t) + q_{02}(t)\oplus B_2^R(t)$$
$$B_1^R(t) = q_{10}(t)\oplus B_0^R(t) + q_{11.3}(t)\oplus B_1^R(t) + q_{16}(t)\oplus B_6^R(t)$$
$$B_2^R(t) = W_2(t) + q_{20}(t)\oplus B_0^R(t) + q_{21.4}(t)\oplus B_1^R(t) + q_{22.5}(t)\oplus B_2^R(t)$$
$$B_6^R(t) = W_6(t) + q_{61}(t)\oplus B_1^R(t) \quad (11)$$

where $W_i(t)$ be the probability that the server is busy in state S_i due to preventive maintenance and repair up to time 't' without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state and so

$$W_1(t) = \{e^{-(\alpha_0+\lambda)t} + (\alpha_0 e^{-(\alpha_0+\lambda)t}\oplus 1)\}\overline{F(t)}$$

$$W_2(t) = \{e^{-(\alpha_0+\lambda)t} + (\alpha_0 e^{-(\alpha_0+\lambda)t}\oplus 1) + (\lambda e^{-(\alpha_0+\lambda)t}\oplus 1)\}\overline{G(t)},$$

$$W_6(t) = \overline{G(t)}$$

taking LT of relations (11). Solving for $B_0^{*p}(t)$ and $B_0^{*R}(t)$, the time for which server is busy due to preventive maintenance and repair respectively are given by

$$B_0^p(t) = \lim_{s\to 0} sB_0^{*p}(t) = \frac{N_1^p}{D},\ \text{and}$$
$$B_0^R(t) = \lim_{s\to 0} sB_0^{*R}(t) = \frac{N_2^R}{D}, \quad (12)$$

Where $N_1^p(t) = W_1^*\left(p_{01}(1-p_{22.5})+p_{02}p_{21.4}\right)$,

$N_2^R(t) = W_2^*\{p_{02}(1-p_{11.3}-p_{16}p_{61})\} - W_6^*\{p_{16}(p_{01}p_{22.5}-p_{01}-p_{02}-p_{21.4})\}$

and D has already mentioned.

Expected Number of Repairs and Preventive Maintenances of the Unit

Let $R_i^R(t)$ and $R_i^P(t)$ be the expected number of repairs and preventive maintenances of the unit by server in time interval (0,t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $R_i^R(t)$ and $R_i^P(t)$ are given as

$$R_0^R(t) = Q_{01}(t)\otimes R_1^R(t) + Q_{02}(t)\otimes R_2^R(t)$$
$$R_1^R(t) = Q_{10}(t)\otimes R_0^R(t) + Q_{11.3}(t)\otimes R_1^R(t) + Q_{16}(t)\otimes R_6^R(t)$$
$$R_2^R(t) = Q_{20}(t)\otimes[1+R_0^R(t)] + Q_{22.5}(t)\otimes[1+R_2^R(t)] + Q_{21.4}(t)\otimes[1+R_1^R(t)]$$
$$R_6^R(t) = Q_{61}(t)\otimes[1+R_1^R(t)]$$

and

$$R_0^p(t) = Q_{01}(t)\otimes R_1^p(t) + Q_{02}(t)\otimes R_2^p(t)$$
$$R_1^p(t) = Q_{10}(t)\otimes[1+R_0^p(t)] + Q_{11.3}(t)\otimes[1+R_1^p(t)] + Q_{16}(t)\otimes R_6^p(t)$$
$$R_2^p(t) = Q_{20}(t)\otimes R_0^p(t) + Q_{22.5}(t)\otimes R_2^p(t) + Q_{21.4}(t)\otimes R_1^p(t)$$
$$R_6^p(t) = Q_{61}(t)\otimes R_1^p(t) \quad (13)$$

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Taking LST of relations (13) and solving for $\tilde{R}_0^R(t)$ and $\tilde{R}_0^p(t)$. The expected number of repairs and preventive maintenances per unit time are respectively given by

$$R_0^R(\infty) = \lim_{s\to 0} s\tilde{R}_0^R(s) = \frac{N_3^R}{D} \tag{14}$$

and $R_0^P(\infty) = \lim_{s\to 0} s\tilde{R}_0^P(s) = \dfrac{N_4^P}{D}$, where

$$N_3^R = p_{02}(1 - p_{11.3} - p_{16}p_{61}) + p_{16}p_{61}[p_{01}(1 - p_{22.5}) + p_{02}p_{21.4}]$$

$$N_4^P = (p_{10} + p_{11.3})\{p_{01}(1 - p_{22.5}) + p_{02}p_{21.4}\}$$

and D has already defined.

Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^P - K_2B_0^R - K_3R_0^R - K_4R_0^P - K_5 \tag{15}$$

K_0= Revenue per unit up-time of the system
K_1= Cost per unit time for which server is busy due preventive maintenance
K_2= Cost per unit time for which server is busy due to repair
K_3= Cost per unit time repair
K_4= Cost per unit time preventive maintenance
K_5= Total installation cost of the system

Conclusion

The trends of some important reliability measures have been observed graphically for the particular case $g(t) = \theta e^{-\theta t}$ and $f(t) = \alpha e^{-\alpha t}$ by taking numerical values of various parameters and costs as shown in figures 2, 3 and 4. It is found that mean time to system failure (MTSF), availability and profit function go on increasing with the increase of preventive maintenance and repair rates. However, their values decline with the increase of failure rate and the rate by which unit under goes for preventive maintenance. The study also reveals that effect of the rate α_0 is more on these measures as compared to the other rates. Hence, a cold standby system of two identical units in which priority is given to repair of one unit over preventive maintenance of the other unit can only be made reliable and profitable to use by repairing the unit in a time less than the time taken for preventive maintenance of the unit. The beauty of the present study is to make a suggestion that performance of a cold standby system can be improved by conducting preventive maintenance after a maximum operation time rather than to spent more money on repair after failure. The application of the present study can be visualized in a water pump system.

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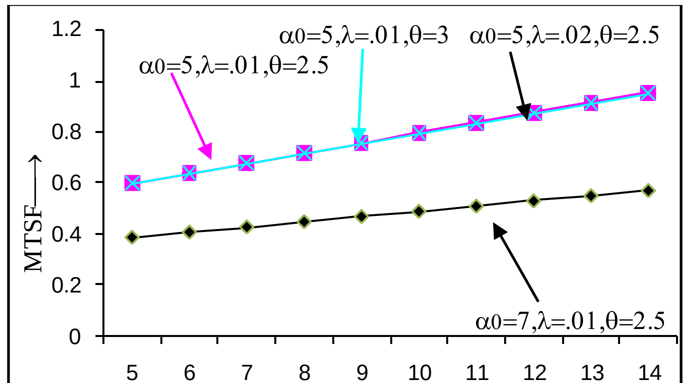

Fig. 2: Preventive maintenance rate(α)→

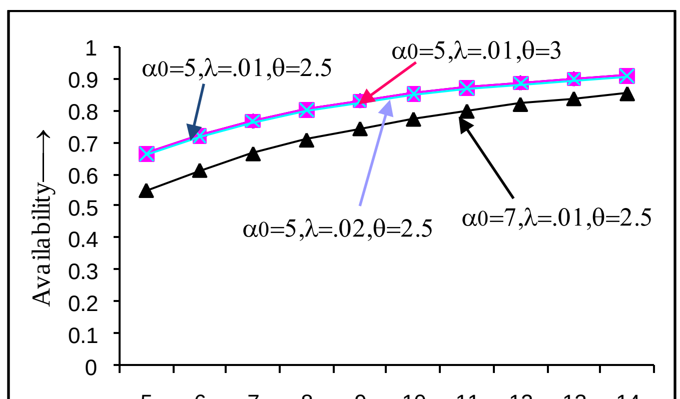

Fig. 3: Preventive maintenance rate(α)→

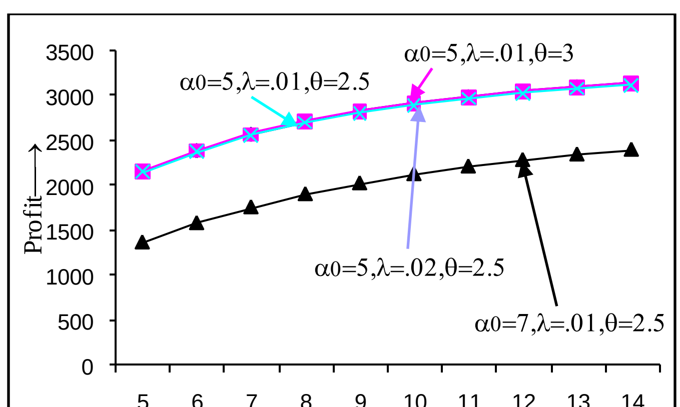

Fig.4: Preventive maintenance rate(α)→

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